Turbulence in Magnetised Plasmas

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rious Non ine r E ects

S Distur nces on n Equi i riu

t t y t t t

$\perp$ $\mathbf{L}_\perp$) $(\mathbf{p}$ $\widetilde{\mathbf{p}}$

Inco pressi e Hydrodyn ics

t t t t t

$$\left(\frac{\partial \mathbf{v}}{\partial \mathbf{t}} \quad \mathbf{v} \quad \nabla \mathbf{v}\right) \qquad - \nabla p$$

t t t t t t ∇ $\mathbf{v}$

$$\left(\frac{\partial}{\partial \mathbf{t}} \quad \mathbf{v} \quad \nabla\right) \nabla \quad \mathbf{v} \quad (\nabla \quad \mathbf{v}) \quad \nabla \mathbf{v}$$

y t t ty

$$t \quad \frac{\partial}{\partial \mathbf{t}} \nabla \quad \mathbf{v} \qquad t \qquad \nabla^2 p \qquad \nabla \quad (\quad \mathbf{v} \quad \nabla \mathbf{v})$$

t t t t t

The C sc de to S er Sc es

t y t t t t t t

y t (t) (t) t

t t y t t

t n t y E v^2= y t t t t t ty (kv)

t t y !

$$(kv) \; {}_{-1} E \; {}_{-1} \quad (kv) \; E \qquad k \qquad k \; {}_{-1}$$

t t t

$(E =k) / k^{-5\ 3}$ ty t t k

t t ty increases t ! t y produced

$$(kv) / k^{2\ 3}$$

Enstrophy in Inco pressi e Hydrodyn ics

t

$$\left(\frac{\partial}{\partial t} \quad v \quad \nabla\right) \nabla \quad v \quad (\nabla \quad v) \quad \nabla v$$

t t ty (t y) t y

$$\frac{\partial W}{\partial t} \quad \nabla \quad (W v) \quad (\nabla \quad v)(\nabla \quad v) \quad \nabla v \qquad W \quad -(\nabla \quad v) \quad (\nabla \quad v)$$

t y t t y t ty t : : :

y t ty t t t ty t t t

t t t t

h t You C n Le rn Just Fro Equ tions

The I port nce of T o Di ension ity

y y
t t t (y t)
t y (t t t t t)

y y y
t t ( ) t A ty $\mathbf{v}_A$
y ty $\mathbf{v}_A^2$
t T M $\mathbf{v}_A^2$ p =B^2

t y t

t t

Equ tions for Be t ves

t

$$\frac{@\quad \mathbf{k}}{@\mathbf{t}} \quad \sum_{-}$$

Energy Tr nsfer

y t y t y y $\mathbf{k}$ t

$$\frac{@\mathsf{U}_{\mathbf{k}}}{@\mathsf{t}} \quad \mathsf{C}_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''$$

$$\frac{@\mathsf{U}_{\mathbf{k}'}}{@\mathsf{t}} \quad \mathsf{C}_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k}$$

$$\frac{@\mathsf{U}_{\mathbf{k}''}}{@\mathsf{t}} \quad \mathsf{C}_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}'$$

t y t t t t t

$$\mathsf{T} \; (\mathrm{k} \quad \mathrm{k}') \quad \mathsf{C}_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''$$

Enstrophy Tr nsfer

t y t y t y y $\mathbf{k}$ t

$$\frac{@W_{\mathbf{k}}}{@t} \quad C_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''$$

$$\frac{@W_{\mathbf{k}'}}{@t} \quad C_{\mathbf{k}\mathbf{k}'} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{k}$$

$$\frac{@W_{\mathbf{k}''}}{@t} \quad C_{\mathbf{k}\mathbf{k}'}$$

The Du C sc de

t y t y t

$$\mathbf{T}(\mathrm{k} \quad \mathrm{k}') \quad \mathbf{C}_{\mathbf{kk}'} \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{C}_{\mathbf{kk}'} \quad \left[(\mathbf{k}'')^2 \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''\right]$$

$$\mathbf{T}(\mathrm{k} \quad \mathrm{k}') \quad \mathbf{C}_{\mathbf{kk}'} \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}'' \quad \mathbf{C}_{\mathbf{kk}'} \quad \left[\mathbf{k}^2(\mathbf{k}')^2 \quad \mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''\right]$$

t t t t t t t $\mathbf{k} \quad \mathbf{k}' \quad \mathbf{k}''$ t t

t t t y t y t **k** t t **k**

y t t t t

y t y t **k hence larger scale**

y

t y t t y t t t y t **W k U k**$^{-1}$

A P ssive Sc r

ty t t t

$$\frac{@\sim}{@t} \quad v \quad r\sim$$

t ~ t y t t t t

k t ty t t t t ty

t t y t y y t

t y t t t y

y t y t

y t ~ $ ~ t t t t

Inco pressi e MHD

t t t y t t

$\Big(\frac{\partial \mathbf{v}}{}$

D Inco pressi e MHD

t t t y t t

$$\left(\frac{@v}{@t} \quad v \quad r\, v\right) \quad r\left(p \quad \frac{B^2}{}\right) \quad \frac{B \quad r\, B}{}$$

t t (r v) v J r B

$$\left(\frac{@}{@t} \quad v \quad r\right) r \quad v \quad \frac{}{c} B \quad r\, J$$

ty t ty t t

$$v \quad v \quad \frac{c}{B^2} B \quad r \qquad \frac{c^2}{B^2} r^2_{\perp} \qquad J_{\parallel} \quad b \quad J$$

t t t ty t

$$\frac{@}{@t} \quad v \quad r \quad b \quad r\, J_{\parallel}$$

pp ic tions of D inco pressi e MHD

y t t u$_\pm$ u b

ty t

$$\mathbf{u} \quad \hat{s} \quad \mathbf{r} \qquad \mathbf{b} \quad \hat{s} \quad \mathbf{r}$$

t ( ) ( ) t A t $(@=@t \; \$ \; \mathbf{v}_A \; \mathbf{r})$

$$\frac{@\mathbf{u}_\pm}{@t} \quad \mathbf{u}_\mp \quad \mathbf{r}\,\mathbf{u}_\pm \qquad \mathbf{r}\, l \quad (\qquad) \, \mathbf{r}_\perp^2 \mathbf{u}_\pm$$

ty t t

$$\mathbf{r}^2 l \quad \mathbf{r} \quad (\mathbf{u}_\mp \quad \mathbf{r}\,\mathbf{u}_\pm)$$

t y y t) y

Dissip tive Coup ing

y ty t t t t t t t t ty

t t

$$E_\| \quad r_\| \tilde{} \quad \frac{}{n\ e} r_\| \tilde{p} \quad {}_\| \tilde{J}_\|$$

ty t t t (t y t ty)

$$\frac{\partial \tilde{p}}{\partial t} \quad v \quad r\,(p \quad \tilde{p}\ ) \quad \frac{T}{e} r_\| \tilde{J}_\|$$

ty t t $\tilde{}$

$$\frac{\partial \tilde{p}}{\partial t} \quad v \quad r\,(p \quad \frac{}{n\ e \quad {}_\| \|} r \left(\tilde{p} \quad n\ e \tilde{} \right)$$

t t t $r_\| \tilde{p}$ n e $r_\| \tilde{}$ t **usual situation** t t

y t t t t y t

Dissip tive Coup ing Mode for E B Tur. u ence

t t t t ! $\mathbf{k}_\perp \mathbf{v}_A$

$$\mathbf{E}_\perp \quad \mathbf{r}_\perp$$

t t t t t t t ty

$$\mathbf{v} \quad \frac{\mathbf{c}}{\mathbf{B^2}}\mathbf{B} \quad \mathbf{r} \qquad \mathbf{r} \quad \frac{\mathbf{c}}{\mathbf{B}}\mathbf{v} \qquad \mathbf{b}$$

t ty t t t t t t

$$\frac{@}{@\mathbf{t}} \quad \mathbf{v} \quad \mathbf{r} \qquad \mathbf{r}_{\|}\mathbf{J}_{\|}$$

t t ::: t t t

$$_{\|}\mathbf{J}_{\|} \quad \frac{}{\mathbf{n}\ \mathbf{e}}\mathbf{r}_{\|}\mathbf{p}$$?(k)Tj /R43 1 Tf 1.58927 017801.2154 -0 102.4T52.58 Tm (k)

Dissip tive Coup ing Mode notes

t t t t

y t t t $ t @B=@t

Sc es in the Dissip tive Coup ing Mode

t t t t t

$$\frac{c^2 M T}{e^2 B^2}\left(\frac{@}{@t} \quad v \quad r\right) r_{\perp}^{2}\frac{e}{T} \quad D\left(\frac{\widetilde{p}}{p} \quad \frac{e}{T}\right)$$

$$\left(\frac{@}{@t} \quad v \quad r\right)\frac{\widetilde{p}}{p} \quad v \quad r \quad p \quad D\left(\frac{\widetilde{p}}{p} \quad \frac{e}{T}\right)$$

t t t y

$$^2 \quad c^2 M T = e^2 B^2$$

t t t $L_{\perp}=c$ t c t $L_{\perp}$

Co put tion Dissip tive Coup ing Mode

t c =L_⊥ y t =L_⊥

$$ ^{-1} e\tilde{} =T \quad p \quad ^{-1}\tilde{p} =p \quad ^{-1}\ ^2 r_\perp^2 \left(e\tilde{} =T\right) $$

y t D DL_⊥=c

$$ \left(\frac{@}{@t}\ \ v\ \ r\right)\ \ D(p\ \ ) $$

$$ \left(\frac{@}{@t}\ \ v\ \ r\right)p\ \ \frac{@}{@y}\ \ D(p\ \ ) $$

t t t t t

$$ v\ \ r\,p\ \ ;p\ \ \frac{@}{@x}\frac{@p}{@y}\ \ \frac{@}{@y}\frac{@p}{@x} $$

t t t (D) t t v @ =@y

I ustr tion of Du C sc de

$(\quad)^2$

y t t t t (t t ` t)

$$p_k(\quad) \quad {}_k(\quad) \quad a_0 \left[\quad (k_\perp^2 = : \quad)^4 \right]^{-1 \; 2} e$$

y

y a_0 t t t :

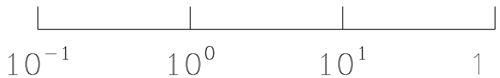
t t y y t D

y t ` ` t p

t t p n

t t y y

A t t t y y p ( )



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Energy Tr nsfer in the Dissip tive Coup ing Mode

t t

t t t y t t t t t ty t t t

t t D : : : : :

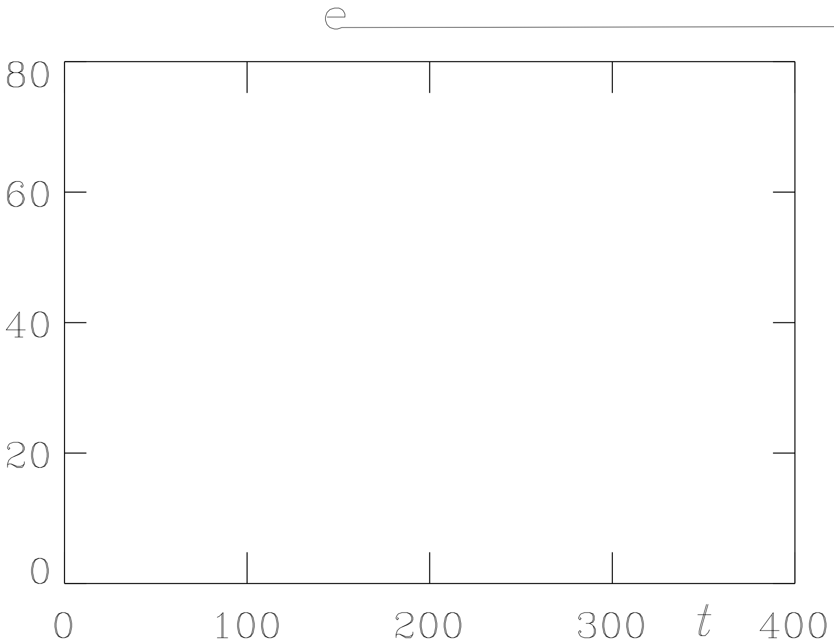
y D :

y y t t D y t t

D ! 1 t t t $J_{||}$! ! p

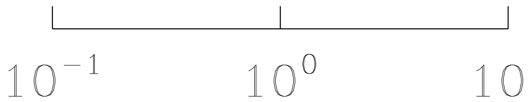
y t v r p t t t $D^{-3\ 4}$

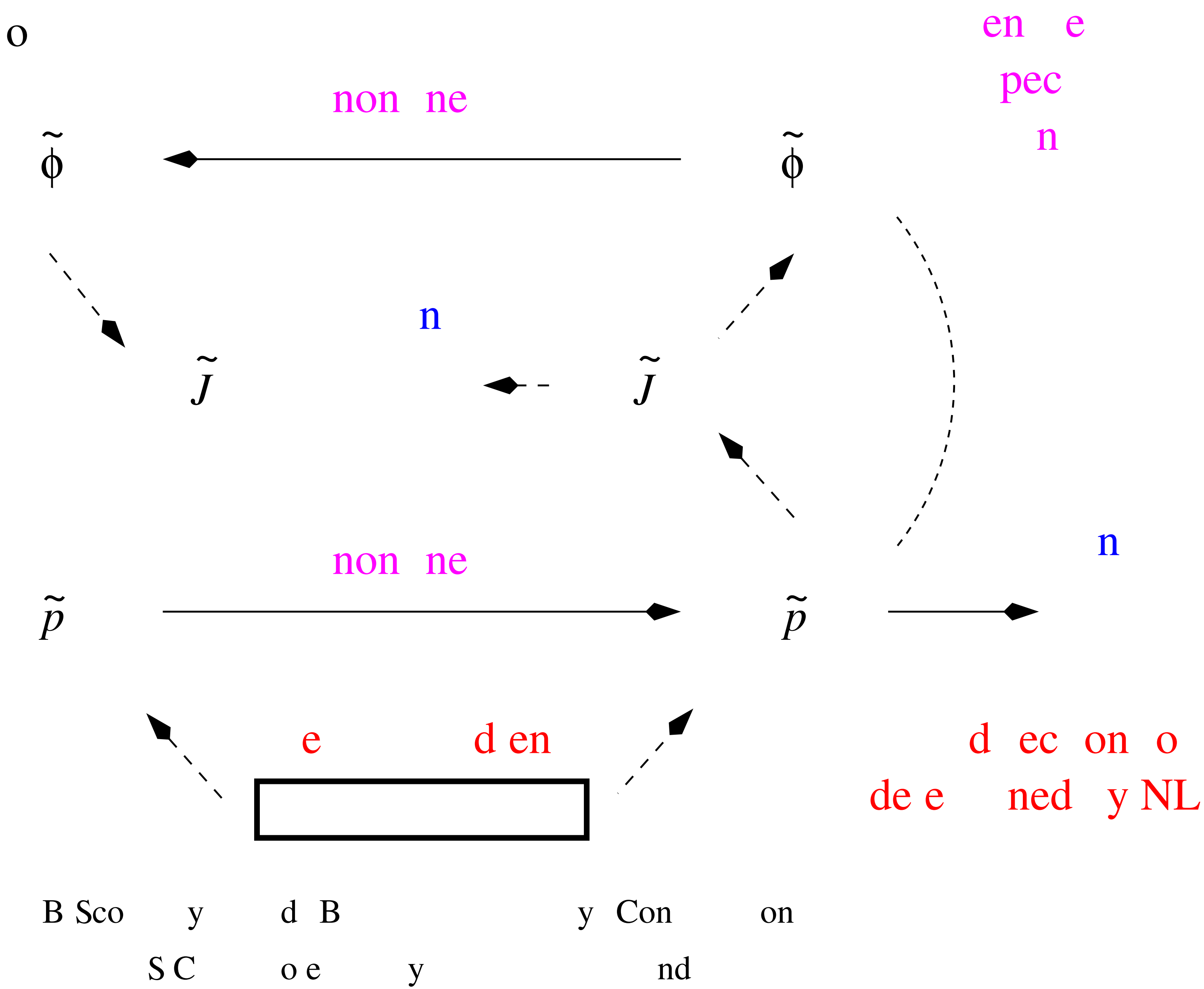
t t t t D :



t t t t t t

y t y t t t t D :




o
non ne
en e
pec
n
n
$\tilde{\phi}$
$\tilde{J}$
$\tilde{p}$
non ne
n
e
d en
d ec on o
de e ned y NL
B Sco y d B y Con on
S C o e y nd

Tr nsport due to E B Tur u ence

t t t t t t :::

$$Q \quad Q \quad Q \quad Q \quad \left\langle -\tilde{p}\ \mathbf{v} \right\rangle \quad Q$$